Chapter III
TRANSPORTATION SYSTEM ANALYSIS

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Lecture Overview

- **Traffic engineering studies**
  - Spot speed studies
  - Volume studies
  - Travel time and delay studies
  - Parking studies

- **Fundamental principles of traffic flow**
  - Traffic flow elements
  - Flow-density relationships
  - Fundamental diagram of traffic flow
  - Mathematical relationships describing traffic flow
  - Shock waves in traffic streams
  - Gap and gap acceptance

- **Queuing Analysis**
  - Queuing Patterns
  - Queuing models

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Fundamental Principles of Traffic Flow

➢ Traffic flow theory involves the development of mathematical relationships among the primary elements of a traffic stream: flow, density, and speed.

➢ Help the traffic engineer in planning, designing, and evaluating the effectiveness of implementing traffic engineering measures on a highway system.

➢ Uses:-
  ➢ To determine adequate lane lengths for storing left-turn vehicles on separate left-turn lanes,
  ➢ The average delay at intersections and freeway ramp merging areas, and
  ➢ Changes in the level of freeway performance due to the installation of improved vehicular control devices on ramps
  ➢ Simulation, where mathematical algorithms are used to study the complex interrelationships that exist among the elements of a traffic stream or network and
  ➢ To estimate the effect of changes in traffic flow on factors such as accidents, travel time, air pollution, and gasoline consumption.
Traffic flow elements

 Flow (q):- is the equivalent hourly rate at which vehicles pass a point on a highway during a time period less than 1 hr. It can be determined by

\[ q = \frac{n \times 3600}{T} \text{ vph} \]

Where: \( n \) = the number of vehicles passing a point in the roadway in \( T \) secs; \( q \) = the equivalent hourly flow.

 Density (k):- sometimes referred to as concentration, is the number of vehicles traveling over a unit length of highway at an instant in time. The unit length is usually 1 mile thereby making vehicles per mile (vpm) the unit of density.

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Traffic flow elements

- **Speed** \((u)\) is the distance traveled by a vehicle during a unit of time.
  - **Time mean speed** \((\overline{u}_t)\) is the arithmetic mean of the speeds of vehicles passing a point on a highway during an interval of time.
    \[
    \overline{u}_t = \frac{1}{n} \sum_{i=1}^{n} u_i
    \]
    Where: \(n = \text{number of vehicles passing a point on the highway; } u_i = \text{speed of the } i\text{th vehicle (ft/sec)}\)
  - **Space mean speed** \((\overline{u}_s)\) is the harmonic mean of the speeds of vehicles passing a point in a highway during an interval of time. It is obtained by dividing the total distance traveled by two or more vehicles on a section of highway by the total time required by these vehicles to travel that distance.
    \[
    \overline{u}_s = \frac{n}{\sum_{i=1}^{n} (1/u_i)} = \frac{nL}{\sum_{i=1}^{n} t_i}
    \]
    Where: \(t_i = \text{the time it takes the } i^{th} \text{ vehicle to travel across a section of highway (sec); } U_i = \text{speed of the } i^{th} \text{ vehicle (ft/sec); } L = \text{length of section of highway (ft)}\)

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Traffic flow elements Cont...

- **The time-space diagram** is a graph that describes the relationship between the location of vehicles in a traffic stream and the time as the vehicles progress along the highway.

- **Time headway** \((t)\) is the difference between the time the front of a vehicle arrives at a point on the highway and the time the front of the next vehicle arrives at that same point.

- **Space headway** \((d)\) is the distance between the front of a vehicle and the front of the following vehicle. It is usually expressed in feet.
Flow-density relationships

- Flow = (density) x (space mean speed)
  \[ q = k \cdot \overline{u}_s \]
- Space mean speed = (flow) x (space headway)
  \[ \overline{u}_s = q \cdot \overline{d} \]
- Density = (flow) x (travel time for unit distance)
  \[ \dot{k} = q \cdot \dot{t} \]
- Average space headway = (space mean speed) x (average time headway)
  \[ \overline{d} = \overline{u}_s \cdot \overline{h} \]
- Average time headway = (average travel time for unit distance) x (average space headway)
  \[ \overline{h} = \overline{t} \cdot \overline{d} \]
Fundamental diagram of traffic flow

- When the flow is very low, there is little interaction between individual vehicles.
- The absolute maximum speed is obtained as the flow tends to zero, and it is known as the mean free speed \( (U_f) \). slopes of lines OB, OC, and OE in Figure 6.3a represents the space mean speeds at densities \( k_b \), \( k_c \), and \( k_e \), respectively.
- The slope of line OA is the speed as the density tends to zero and little interaction exists between vehicles. The slope of this line is therefore the mean free speed \( (U_f) \).
Mathematical relationships describing traffic flow

- Mathematical relationships describing traffic flow can be classified into

- **The macroscopic approach:** Considers traffic streams and develops algorithms that relate the flow to the density and space mean speeds.

- **Green shields Model.** Green shields carried out one of the earliest recorded works, in which he studied the relationship between speed and density. He hypothesized that a linear relationship existed between speed and density.

- **Greenberg Model.** Use the analogy of fluid flow to develop macroscopic relationships for traffic flow.

- **The microscopic approach,** which is sometimes referred to as the car-following theory or the follow-the-leader theory, considers spacing between and speeds of individual vehicles.
Green shields Model

- He hypothesized that a linear relationship existed between speed and density, which he expressed as \( \bar{u}_s = u_f - \frac{u_f}{k_j} \).

- Since \( q = \bar{u}_s k \), there for \( \bar{u}_s^2 = u_f \bar{u}_s - \frac{u_f}{k_j} \).

- Also substituting \( k/q \) for \( \bar{u}_s \) gives \( q = u_f k - \frac{u_f}{k_j} k^2 \).

- Differentiating \( q \) with respect to \( \bar{u}_s \), we obtain \( 2\bar{u}_s = u_f \frac{du_f}{dq} \).

- For maximum flow, \( \frac{dq}{d\bar{u}_s} = 0 \Rightarrow k_j = 2\bar{u}_s \frac{k_j}{u_f} \Rightarrow u_o = \frac{u_f}{2} \).

- Differentiating \( q \) with respect to \( k \), we obtain for maximum \( q \) \( \frac{dq}{dk} = 0 \Rightarrow u_f = 2k \frac{u_f}{k_j} \Rightarrow k_o = \frac{k_j}{2} \).

- The maximum flow can therefore be \( q_{\text{max}} = \frac{k_j u_f}{4} \).

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Greenberg Model

- Using the fluid-flow analogy was developed by Greenberg in the form

\[ \bar{u}_s = c \ln \frac{k_j}{k} \quad q = ck \ln \frac{k_j}{k} \]

- Differentiating q with respect to k, we obtain

\[ \frac{dq}{dk} = c \ln \frac{k_j}{k} - c \]

- For maximum flow, \( \frac{dq}{dk} = 0 \), \( \ln \frac{k_j}{k} = 1 \)

- Giving \( \ln k_j = 1 + \ln k_o \)

- That is, \( \ln \frac{k_j}{k_o} = 1 \) and Substituting 1 for \( \ln \frac{k_j}{k_o} \) gives \( u_o = c \)

- Thus, the value of c is the speed at maximum flow.
Example

1.1 Find the equation of the following relationship
   • linear relationship \( Y=ax + b \)
   • logarithmic relationship \( Y=a*\ln(x) + b \)

1.2. Transform these formulas to show the model of Greenshields and Greenberg and find \( V_m, V_f, K_m \) and \( K_j \).

1.3. Find \( k = k(u) \), \( q = q(u) \) and \( q = q(k) \)

1.4. Make a graph of \( v=v(k) \), \( q=q(v) \) & \( q(k) \)

<table>
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<tr>
<th>Data set</th>
<th>Speed ( u ) (km/hr)</th>
<th>Concent. ( K ) (veh/km)</th>
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<tr>
<td>1</td>
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<td>13</td>
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</tbody>
</table>
Microscopic Approach

- sometimes referred to as the car-following theory or the follow-the-leader theory, considers spacing between and speeds of individual vehicles.

- Consider two consecutive vehicles, A and B, on a single lane of a highway, as shown in the Figure. If the leading vehicle is considered to be the nth vehicle and the following vehicle is considered the (n + 1)th vehicle, then the distances of these vehicles from a fixed section at any time t can be taken as \( x_n \) and \( x_{n+1} \) respectively.

- If the driver of vehicle B maintains an additional separation distance \( P \) above the separation distance at rest \( S \) such that \( P \) is proportional to the speed of vehicle B, then

\[
p = \rho \cdot \dot{x}_{n+1}
\]

Where: \( \rho \) = factor of proportionality with units of time; \( \dot{x}_{n+1} \) = speed of the (n + l)th vehicle we can write

\[
x_n - x_{n+1} = \rho \cdot \dot{x}_{n+1} + S.
\]

- Where \( S \) is the distance between front bumpers of vehicles at rest

Differentiating

\[
\dot{x}_{n+1} = \frac{1}{\rho} (\dot{x}_n - \dot{x}_{n+1})
\]
Microscopic Approach Cont…

\[ x_{n+1} \]

\[ P = \rho \dot{x}_{n+1} \]

\[ S \]

\[ x_n \]

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Shock waves in traffic streams

- The sudden reduction in capacity due to accidents, reduction in the number of lanes, restricted bridge sizes, work zones, a signal turning red, and so forth, creating a situation where the capacity on the highway suddenly changes from $C_1$ to a lower value of $C_2$, with a corresponding change in optimum density from $k_o^a$ to $k_o^b$, a value of

- The point at which the speed reduction takes place can be approximately noted by the turning on of the brake lights of the vehicles.

- An observer will see that this point moves upstream as traffic continues to approach the vicinity of the bottleneck, indicating an upstream movement of the point at which flow and density change. This phenomenon is usually referred to as a shockwave in the traffic stream.
Shock waves in traffic streams Cont…

- Let us consider two different densities of traffic, $k_1$ and $k_2$, along a straight highway, where $k_1 > k_2$. Let us also assume that these densities are separated by the line $w$, representing the shock wave moving at a speed $U_w$.

- With $U_1$ equal to the space mean speed of vehicles in the area with density $k_1$ (section P), the speed of the vehicle in this area relative to the line $w$ is

$$u_{r1} = (u_1 - u_w)$$

- The number of vehicles crossing line $w$ from area P during a time period $t$ is

$$N_1 = u_{r1} k_1 t$$

- Similarly, the speed of vehicles in the area with density $k_2$ (section Q) relative to $w$ is

$$u_{r2} = (u_2 - u_w)$$

- And the number of vehicles crossing line $w$ during a time period $t$ is

$$N_2 = u_{r2} k_2 t$$

- Since the net change is zero

$$N_1 = N_2$$

$$(u_1 - u_w) k_1 t = (u_2 - u_w) k_2 t$$

$$u_2 k_2 - u_1 k_1 = u_w (k_2 - k_1)$$

$$q_2 - q_1 = u_w (k_2 - k_1)$$

$$u_w = \frac{q_2 - q_1}{k_2 - k_1}$$

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Example

Numerically solve the example problem shown in figure below if flow states A and B are defined as follows: $u_A$ and $u_B$ are to equal to 30 and 40 miles per hour, respectively, and $k_A$ and $k_B$ are equal to 48 and 24 vehicles per mile per lane, respectively. How many vehicles leave flow states B in a 1-hour period?
Gap and gap acceptance

- Gap acceptance: The evaluation of available gaps and the decision to carry out a specific maneuver within a particular gap.
- Important concept of traffic flow if there is interaction of vehicles as they join, leave, or cross a traffic stream.
  - Examples of these include ramp vehicles merging onto an expressway stream, freeway vehicles leaving the freeway onto frontage roads, and the changing of lanes by vehicles on a multilane highway.

Following are the important measures that involve the concept of gap acceptance.

- **Merging** is the process by which a vehicle in one traffic stream joins another traffic stream moving in the same direction, such as a ramp vehicle joining a freeway stream.
- **Diverging** is the process by which a vehicle in a traffic stream leaves that traffic stream, such as a vehicle leaving the outside lane of an expressway.
- **Weaving** is the process by which a vehicle first merges into a stream of traffic, obliquely crosses that stream, and then merges into a second stream moving in the same direction; for example, the maneuver required for a ramp vehicle to join the far side stream of flow on an expressway.

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Gap and gap acceptance Cont…

- **Gap** is the headway in a major stream, which is evaluated by a vehicle driver in a minor stream who wishes to merge into the major stream. It is expressed either in units of time (time gap) or in units of distance (space gap).
- **Time lag** is the difference between the time a vehicle that merges into a main traffic stream reaches a point on the highway in the area of merge and the time a vehicle in the main stream reaches the same point.
- **Space lag** is the difference, at an instant of time, between the distance a merging vehicle is away from a reference point in the area of merge and the distance a vehicle in the main stream is away from the same point.
- **Gap acceptance:** evaluate the gaps that become available to determine which gap (if any) is large enough to accept the vehicle, in his or her opinion.
Thank You!